Energy

**Problem**

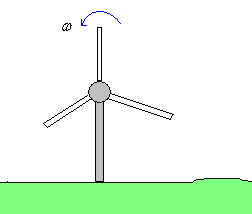
Consider the disk above again. If it has mass 25kg, and is rotating at a rate of 3 rad/s, what is its KE?

Well, I = (1/2)MR2 = (1/2)(25)(1.3)2 = 21.1, and so,



**Problem**

Wind farms are one way to generate power. Suppose a windmill with three prongs has a radius of 70m. Further suppose that each weighs 10 000kg. If the windmill is turning at a rate of 10 revolutions per minute, what rotational kinetic energy is stored in the windmill?



We will use the formula,



To calculate I we will treat the windmill as three boards rotating about their endpoints. Each such board would have a moment of inertia: Ib = (1/3)mℓ2. So altogether the windmill would have a moment of inertia: Iw = 3(1/3)mℓ2 = mℓ2. Further, ω = 10 rev/min. = 10(2π)/60 rad/s = π/3 rad/s. So filling these in we have,



Work-Energy equation w/ rotation

**Problem**

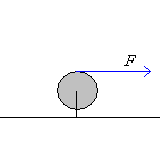
As you ride a stationary bike you apply a torque to the wheel which varies with angular position according to the formula: τ(θ) = 3sin2­θ. What is the angular velocity of the wheel after 10 rotations if its moment of inertia is I = 10kg·m2? Note the identity sin2θ = [1-cos(2θ)]/2 might be useful (it probably is).

Use the WE equation,



**Problem**

Suppose that a grindstone, which we’ll approximate as a solid disk, is initially at rest. Then suppose we exert a constant force of 25N tangent to the edge of the grindstone (as illustrated below). After the grindstone completes 20 revolutions, what will be its angular velocity? Assume the grindstone has a mass of 5kg, and a diameter of 50cm.

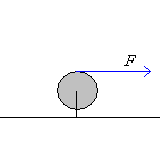


Use the WE equation,



**Example**

If the disk above has a radius of 1.3m, and the applied force is 15N. What work does the force do in rotating it through an angle of 400˚?



To answer, we simply form,



The magnitude of the angle is:



The torque is:

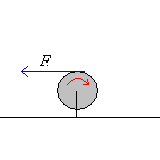


To calculate the work, we need τ||, and this is 18 because all of the torque is in the direction of the angular displacement (both are into the page). So the work done is:



**Example**

If the disk is rotating at a rate of 3 rad/s. What F must be applied to slow it down to rest in 5 rev.?



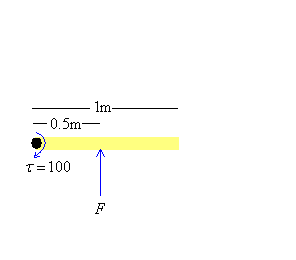
We apply the WE equation. The work done on the disk is that done by the torque supplied by F. The energy of the disk is purely rotational kinetic energy.



**Example: Pushing open a door**

Suppose you’re trying to push open a door of mass 50kg, width 1m, and height 2.5m. If the hinges exert a counter torque of τhinge = -100N·m **z**. If you apply a force of 220 N, how fast will the door be rotating by the time you’ve pushed it through an angle of 30 degrees?

The situation is illustrated, top-down, below:



We can apply the WE equation to the door.



Since the hinge force and F-force are non-conservative, and since the change in energy of the door is purely kinetic, we have,

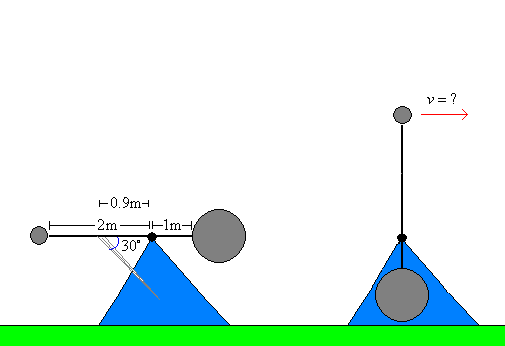


So,



**Example: Trebuchet**

Consider the trebuchet, a type of catapult looking device used during the middle ages to besiege enemy castle, and in modern times to hurl pumpkins and the like: Suppose the counterweight is 500kg, and the projectile is 10kg. If we cut the ropes, what will be the speed of the projectile when released?



We can use the WE equation for rotational motion.



Now what are the forces acting on our system? There is gravity of course, and also the pivot point will exert a force Fp, if the pivot isn’t frictionless. As the arm rotates, Fp will also exert some torque on the arm which will consequently do some work on the arm as it rotates. But for simplicity sake, we’ll assume that this work is negligible. Then we have,



Now what is I? Well, for the first, we will be consider the rotation about the pivot point. So that is where O will be, and we’ll have,



And now we’re ready to work it all out,

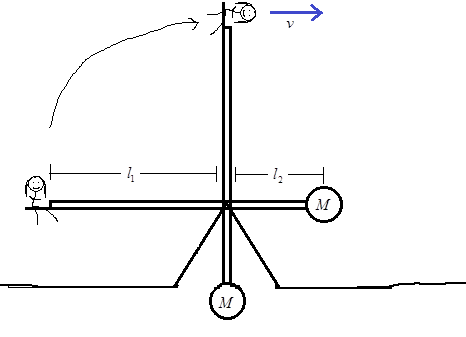


What speed does this correspond to, as far as the projectile is concerned? Well it would be,



**Problem 5**

Since the last situation was so much fun, you decide to do this again. This time you construct a trebuchet. A counterweight (M = 550kg) is at one end a distance ℓ2 = 1.5m away from the pivot. The child (m = 10kg) is a distance ℓ1 = 3m away from the pivot. The mass of the uniform rod connecting the two is mrod = 35kg. When released from the horizontal position, what will be the child’s velocity when she gets to the top? Note that this problem is similar to that done in class, but there are two extra things to take into account: the rotational kinetic energy of the rod (breaking it up into two rods ℓ1 and ℓ2 might help here), and the gravitational work done on the rod. This will be Wg = -mgΔy as usual, but where Δy is the vertical displacement of the *center* of the rod.



We’ll split the rod into two sections of length ℓ1 and ℓ2. Note that the mass of the first rod will be m1 = (35kg)∙ℓ1/(ℓ1 + ℓ2) = 23.3kg. The second part will have mass m2 = 35kg – 23.3kg = 11.7kg. Applying WE equation to the child + rod1 + rod2 + M, we get:



Now we will use the fact that vchild = ωℓ1 = 3ω, and vM =ωℓ2 = 1.5ω to write:

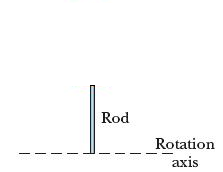


and so the speed of the child is:



**Problem 1**

The figure shows a rigid rod (of length *L* = 2m and also of mass *m* = 0.30 kg). The rod is upright, but we nudge it so that it rotates around a horizontal axis in the plane of the rod, through the lower end of the rod. Assuming that the energy given to the rod in the nudge is negligible, what is the rod’s angular speed about the rotation axis when it passes through the upside-down (inverted) orientation?



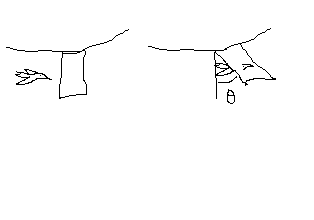
Using Work energy equation,



and so we find,



**Question 6.** A 75g bird flying at 20m/s becomes embedded in the middle of a very starchy towel (i.e. stiff like a board). Say the towel has a mass m = 1.2kg, and length 80cm. What maximum angle does the shirt + bird rotate through?



Applying conservation of angular momentum to the collision we have:



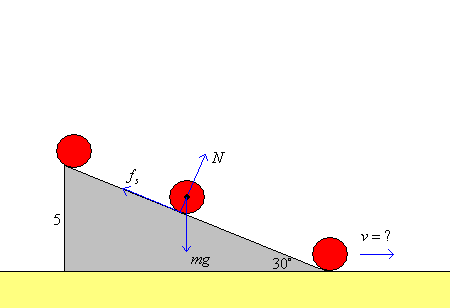
Then to determine the max angle we use conservation of energy:



Work-Energy Equation w/ rotation & translation

**Example: Ball rolling down a hill again**

Let us calculate how long it will take a basketball to roll down a hill of height 5m and incline 30˚. Suppose the ball has a mass 0.5kg, and radius R = 10cm. We already did this problem last lecture, but now we’ll do it with the energy approach.



Our system will be the ball. Between the top and bottom, we have,



So what are the non-conservative forces on the ball. They are simply fs. What is the work that it does on the ball? It is 0 in fact. This is because the ball isn’t moving at the point of contact between the ball and the surface, and so d**r** = 0 (for that point). And thus fs doesn’t do any work. So we have conservation of energy,



Now v = ωR, and I = (2/5)mR2, so…

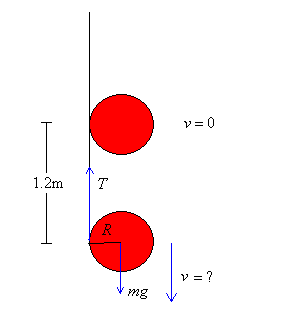


which is what we found before when we did this problem strictly with N2L.

**Example: Yo-yo**

Suppose you have a 0.25kg yo-yo shaped like a disk with diameter 5cm. If you let it unwind down the string a distance of 1.2m, what will be its velocity?

First we draw in all forces, done below,



Then we could use N2L along with the torque equations to calculate the acceleration, and from that find the velocity, as we did previously. But instead we’ll use the WE equation.



The only non-conservative force is the tension. Tension does work on the disk and on the ball. But these works cancel out. To see this, consider that the net work done on the ball + disk is:



because the distance the ball drops, Δy is equal to the arc length that the wheel unravels through. So we have,



Now use the fact that v = ωR, and substitute into the second term,

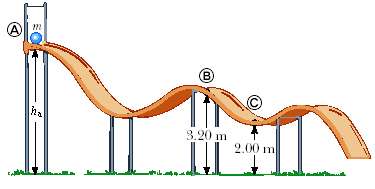


Filling in the details, we get,



**Problem**

Now suppose that that the bowling ball *rolls* down the slide from the height ha = 10m. What will be its velocity at point C now?



We again use the WE equation,



Still the non-conservative work is 0. But now we have to include rotational kinetic energy. So we got,

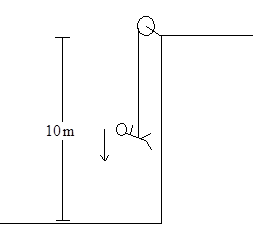


Now use the fact that for a rolling object, its rate of rotation ω is equal to its velocity, v, divided by its radius R. So we have ω = v/R. Also use the fact that for a sphere rotating about its own axis we have I = (2/5)mR2. Filling these in we get,



**Problem**

Suppose you drop off a rock wall, your rope attached to a pulley, shaped like a disk. As you fall, the rope will spin the disk (like a fish pulling on the reel of a fishing pole) which will in turn slow your rate of decent. Suppose the mass of the disk is M = 100kg. And suppose your mass is m = 70kg. If you drop from rest at the top of the wall, what will be your speed when you hit the floor?

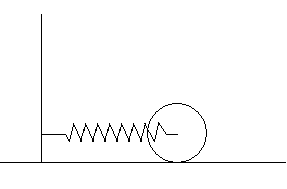


Using WE equation,



**Problem 8.**

Consider the following toy – a 0.75kg disk connected to a spring. The spring constant is k = 120N/m. Suppose you take the disk and stretch it out a distance *x* = 24cm from its equilibrium point. When you let it go, it will roll forward. What will be its speed when it crosses its equilibrium point again? Don’t forget to include rotational and translational kinetic energy in your analysis.



Using WE equation:



**Problem 4**

Consider a cylinder with radius, and a sphere with radius R, both at the top of a hill of height h. If you let them roll down the hill, which will get there first, assuming they have the same mass?

**Solution**

We’ll answer the question by determining the velocity of each at the bottom of the hill. The faster one will have gotten there first. So using the work-energy equation on the cylinder we have,



There will be no non-conservative on the cylinder. Remember that the force of friction will do no net work on rolling objects. So we’ll get,



I for a cylinder is (1/2)mR2, and also v = ωR. Plugging these in we get,



Now do the same for the sphere, using its moment of inertia I = (2/5)mR2,



since 10/7 > 4/3, the sphere would be going faster at the bottom of the hill, and therefore will get there faster.

**Problem 5**

Suppose you have a mass of 65kg, your bike frame has a mass of 10kg, and the tires on your bike each have a mass of 5kg, and a diameter of 80cm. If you coast down a hill 15m high, how fast will you be going at the bottom?

**Solution**

We use the Work energy equation again, of course,



There is no non-conservative work being done on you+bike, since friction cancels out again, and we’ll neglect wind resistance. So we’ll have,



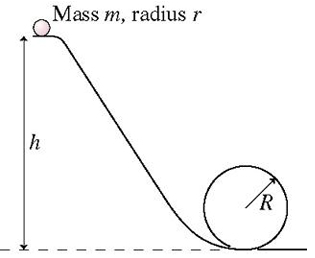
where mp is the mass of the person, mb is the mass of the bike frame, and mw is the mass of the wheel (there are two wheels). Note that we also have rotational kinetic energy of the rotating wheels. We’ll approximate the wheels as two hoops. So their moments of inertia will be I = mr2. Finally, ω = v/R. So plugging these in,



Plugging in all the numbers we get,



**Question 5**. A marble rolls down the track and around a loop-the-loop of radius *R = 80 cm*. The marble has mass *m* and radius *r* = *1* *cm* What minimum height *h* must the track have for the marble to make it around the loop-the-loop without falling off?



First, we can calculate the velocity the ball must have in order to not fall off the track at the top. This comes from N2L for circular motion:



Being careful about heights and distances and such…since there is no work done by friction, energy is conserved. Let’s compare energy at top of ramp and then at top of circle:



**Question 11.** A solid sphere is rolling along the floor at a speed of 12m/swhen it comes to an incline. How high, y, up the incline will it roll, before turning around?

We have, from the WE equation: Ei + Wn.c. = Ef. Again Wn.c. = 0, since the only non-conservative forces acting on the object are static friction and the normal force – neither of whch do work. So we have Ei = Ef,

